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ABSTRACT

Flexible mechanical metamaterials are compliant structures engineered to achieve unique properties via the large deformation of their components. While their static character has been studied extensively, the study of their dynamic properties is still at an early stage, especially in the nonlinear regime induced by their high deformability. Nevertheless, recent studies show that these systems provide new opportunities for the control of large amplitude elastic waves. Here, we summarize the recent results on the propagation of nonlinear waves in flexible elastic metamaterials and highlight possible new research directions.

I. INTRODUCTION

Over the last two decades, metamaterials—materials whose properties are defined by their structure rather than their composition—have been a real magnet for scientists, generating significant interest in the research community.[1–4] While initial efforts focused on metamaterials that manipulate electro-magnetic,[1] acoustic,[3,4] or thermal[5] properties, in recent years, the concept has also been extended to mechanical systems.[6–8] Ongoing advances in digital manufacturing technologies[9–12] have stimulated the design of mechanical metamaterials with highly unusual properties, including negative Poisson's ratio,[13,14] negative thermal expansion,[15,16] and negative compressibility[17] in the static regime, as well as low-frequency spectral gaps,[18,19] negative dynamic properties,[20] and advanced dispersion effects[21] in the dynamic regime. Furthermore, it has been shown that large deformations and mechanical instabilities can be exploited to realize flexible mechanical metamaterials (flexMMs) with new modes of functionality.[6] The complex and programmable deformation of flexMMs make them an ideal platform to design reconfigurable structures[22] as well as soft robots[23] and mechanical logic devices.[24–26] Furthermore, they also provide opportunities to manipulate the propagation of finite amplitude elastic waves. Differently from granular media whose nonlinear response is determined by the contacts between grains,[27–29] the nonlinear dynamic response of flexMMs is governed by their architecture. By carefully choosing the geometry, a flexMM can be designed to be either monostable or multistable or to support large internal rotations—all features that have been shown to result in interesting nonlinear dynamic phenomena.

In this Perspective article, we first review the nonlinear dynamic effects that have been recently reported for flexMMs: the propagation and manipulation of vector elastic solitons, rarefaction solitons, and topological solitons (also referred to as transition waves). We then describe the numerical and analytical tools that are typically used to investigate the propagation of these nonlinear waves. Finally, we outline the key challenges and opportunities for future work in this exciting area of research.

II. NONLINEAR DYNAMIC EFFECTS IN FLEXMM

While nonlinear elastic waves in engineered materials have mostly been experimentally studied in granular media,[27–32] flexMM also provide an ideal environment for their propagation, since they can support a wide range of effective nonlinear behaviors that are determined by their architecture. By carefully tuning these nonlinear behaviors, novel dynamic effects have been demonstrated. First, metamaterials based on the rotating-square mechanism have been shown to support the propagation of elastic vector solitons—solitary pulses with both translational and rotational components, which are coupled together and copropagate without distortion or splitting due to the perfect balance between dispersion and nonlinearity.[33–35] Second, since flexMM can typically support tensile deformation, the propagation of rarefaction solitons has also been demonstrated.[36–39] Third, by designing their energy landscape to be multiwelled, it has been shown that they can support the propagation of topological solitons (also referred to as transition waves)—nonlinear pulses that sequentially switch the structural elements from one stable state to another.[24,40–43]

A. Elastic vector solitons

Flexible metamaterials comprising a network of squares connected by thin and highly deformable ligaments [see Figs. 1(a) and 1(b)] have long attracted significant interest due to their effective negative Poisson's ratio[46–48] and their support of buckling-induced pattern transformations.[14,49] Additionally, it has recently been shown via a combination of experiments and analyses that the nonlinear dynamic response of these structures is also very rich.[33–35,50] First, it has been demonstrated that even one-dimensional (1D) chains of these metamaterials support the propagation of elastic vector solitons with both translational and rotational components which are coupled together and copropagate without dispersion.[33,34] The existence of these pulses is enabled by the perfectly balanced dispersive and nonlinear effects.[51] While vector solitons have been previously reported in optics,[52] their observation in networks of hinged squares is the first for the elastic case. Importantly, the vectorial nature of such solitons gives rise to a vast array of exotic mechanical phenomena. For example, due to the weak coupling between their two components, at small enough amplitudes, the vector solitons become dispersive and fail to propagate.[34] Furthermore, the vectorial nature of the supported solitons leads to anomalous collisions.[50] While, as expected, the solitons emerge unaltered from the collision if they excite rotations of the same direction, they do not penetrate each other and instead repel one another if they induce rotations of the opposite direction. Finally, it has been shown that nonlinear propagation in two-dimensional (2D) systems of rotating squares exhibit very rich direction-dependent behaviors such as the formation of *sound bullets* and the separation of pulses into different solitary modes.[35] As such, these studies suggest that flexible metamaterials based on the rotating-square mechanism may represent a powerful platform to manipulate the propagation of nonlinear pulses in unprecedented ways.

B. Rarefaction solitons

Granular systems that derive their nonlinearity from Hertzian contact are well known to support the propagation of compressive solitons in many instances.[53–59] Yet, it is challenging for such systems to support rarefaction solitons due to their lack of tensile cohesion. While it has been shown via a combination of theoretical and numerical analyses that a precompressed discrete chain with strain-softening interactions could support rarefaction solitons,[60] experimental demonstration of this behavior has remained elusive due to the challenges in fabricating an effective strain-softening mechanism. By contrast, flexMMs can be easily designed to support softening nonlinearity under compression and, therefore, rarefaction solitons, an effect that can, in principle, allow useful applications by enabling efficient impact mitigation. This is the case for a 1D array of buckled beams[36] [Fig. 1(c)], as well as a chain of triangulated cylindrical origami[37] [Fig. 1(d)]. Both systems exhibit effective strain-softening behavior and have been shown to support the propagation of rarefaction solitons. Furthermore, rarefaction solitons have been predicted but not yet experimentally observed in tensegrity structures[38] [Fig. 1(e)] and statically compressed metamaterials based on the rotating-square mechanism.[61] Note that, for the latter, vector rarefaction solitons are predicted to propagate, sharing the interesting features discussed in Sec. II A.

Beyond impact mitigation, rarefaction solitons have also been harnessed to enable locomotion in a slinky-based soft robot[39] [see Fig. 1(f)]. The nondispersive nature and compactness of the solitary pulses make them extremely efficient in transferring the energy provided by the actuator to motion, ultimately resulting in an efficient pulse-driven locomotion.

C. Topological solitons/transition waves

In addition to vector and rarefaction solitons, another category of nonlinear wave, comprising what are called transition waves or topological solitons, has also received a significant amount of recent attention. These waves represent moving interfaces that separate regions of different phases and play a major role in a wide range of physical phenomena, including damage propagation in solids,[62] dynamic phase transitions,[63–65] and phase transformations in crystalline materials.[66–69] Recently, it has been shown that transition waves can also propagate in flexMM made with elements possessing multi-well energy landscapes, with each energy well corresponding to a stable spatial configuration. When a transition wave propagates in these systems, it can be visualized as a solitary pulse that sequentially switches the individual units of the metamaterial from one stable configuration to another.[24,40,42,43,70–74]

Transition waves were first experimentally observed in flexMM in a system comprising a 1D array of bistable and prestressed composite shells coupled magnetically[40] [Fig. 1(g)]. More specifically, the shells are designed to have two energy minima of different heights. Therefore, the transition between the two stable states involves a net change in stored potential energy, which, depending on the direction of the transition, either absorbs energy or releases stored potential energy. If the bistable shells are initially set to their higher-energy stable configuration, a sufficiently large displacement applied to any of them can cause the element to transition states, producing a nonlinear transition wave that propagates

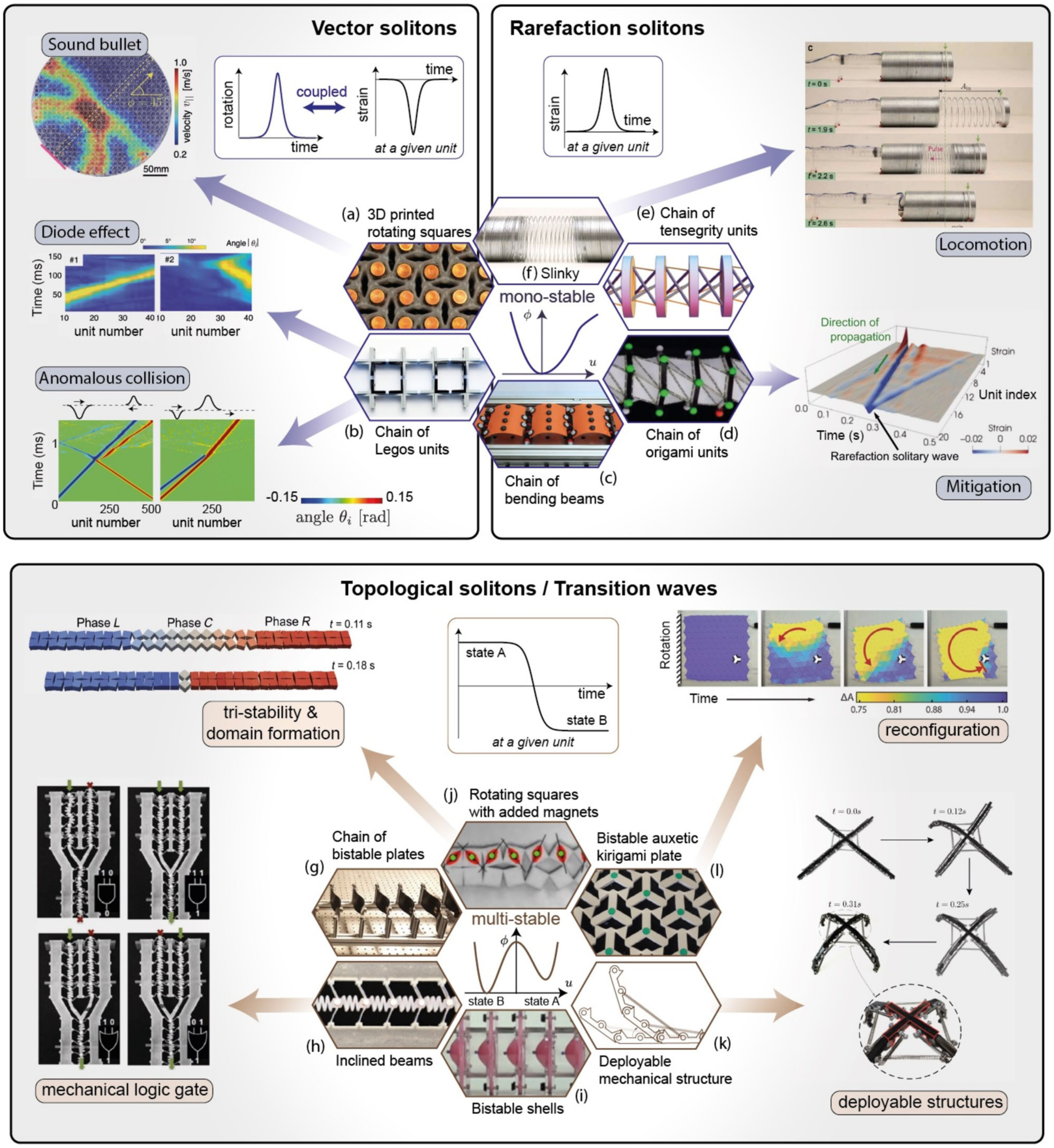

FIG. 1. FlexMMs provide a rich platform to manipulate the propagation of nonlinear waves. Vectors solitons have been observed in (a) a 2D flexMM based on the rotating-square mechanism,[33] (b) a chain of Lego units connected by flexible hinges.[34] Rarefaction solitons have been observed in (c) a chain of hinged buckled beams,[36] (d) a chain of origami units [graph and picture licensed under a Creative Commons Attribution (CC BY-NC) license, reproduced and cropped from Ref. 37], (e) a chain of tensegrity units (reproduced from Ref. 38 with the permission of AIP Publishing), and (f) a Slinky.[39] Topological solitons (transition waves) have been observed in (g) a chain of bistable plates coupled by the magnetic force[40,44] [picture licensed under a Creative Commons Attribution (CC BY) license, reprinted and cropped from Ref. 44], (h) a chain of bistable inclined beams coupled by elastic elements,[24] (i) a chain of bistable shells coupled by pressurized air,[41] (j) a system of rotating squares with embedded magnets,[45] (k) a 1D linkage,[42] and (l) a 2D multistable kirigami structure.[43]

indefinitely outward from the point of initiation with constant speed and shape. Furthermore, transition waves are not sensitive to the specific signal that triggers them and can be initiated by any input signal of sufficient amplitude. Such robustness has been recently harnessed to concentrate, transmit, and harvest energy independently from the excitation[44] [see Fig. 1(g)]. Specifically, the energy carried by the transition waves has been focused and subsequently harvested in lattices by introducing engineered defects and integrating electromechanical transduction.[44]

Interestingly, because of the energy released upon transition between states by each element, stable and long-distance propagation of transition waves in multistable systems is possible even in the presence of significant dissipation[24]—a feature that has been demonstrated for a soft structure composed of elastomeric bistable beam elements connected by elastomeric linear springs. Such ability to transmit a mechanical signal over long distances with high fidelity and controllability has been shown to provide opportunities for signal processing, as demonstrated by the design of soft mechanical diodes and logic gates[24] [see Fig. 1(h)]. Notably, these systems have been also recently realized at the micro-scale using two-photon stereolithography,[75] providing a first step toward mechanical chips. However, it is important to note that, while bistable unit cells with two stable states of different energy levels enable long-distance propagation of transition waves, they inherently prevent bidirectional signal transmission.[40] Furthermore, they require an external source of energy to be provided to reset them to their higher-energy state if additional propagation events are desired.

Bidirectional propagation of transition waves can be achieved by utilizing bistable elements that possess equal energy minima[41] [Fig. 1(i)]. However, since such bistable elements do not release energy when transitioning between their two stable states, the distance traveled by the supported transition waves is limited by unavoidable dissipative phenomena. To overcome this limitation, two strategies have been proposed. On the one hand, it has been shown that the propagation distance of transition waves can be extended by introducing elements with tunable energy landscape, since they can be easily set to release the energy required to compensate for dissipation.[41] On the other hand, long-distance propagation of transition waves has been demonstrated in a 1D array of bistable elements with monotonically decreasing energy barriers[76] but such a gradient in the energy landscape prevents bidirectionality.

FlexMM can also be designed to possess more than two energy minima [Fig. 1(j)]. Just as in the bistable systems described above, transition waves can propagate when a transition from one stable well to another is initiated. However, since multiple types of energetically-favorable transitions are possible (e.g., a system in a higher energy well might support transition waves to two different lower energy wells, each associated with distinct spatial configurations), incompatible transition waves can propagate and collide, leading to non-homogeneous spatial configurations. For example, transition waves have been demonstrated in rotating-square systems with permanent magnets added to the faces.[45] In contrast to the buckled elements described above, each unit in the metamaterial supports up to three stable configurations, enabled by the ability of the squares to be stable in "open," "clockwise," or "counterclockwise" configurations. The ability of multistable systems to support the formation of many configurations of stationary domain walls could allow the design of transformable mechanical metamaterials that can be reversibly tuned across a large range of mechanical properties.

Finally, while all initial studies on the propagation of transition waves in flexMM have considered 1D chains, recently transition waves have also been studied in flexMMs with higher dimensions. As a first step in this direction, the response of a network of 1D mechanical linkages that supports the propagation of transition waves has been investigated[42] [Fig. 1(k)]. It has been shown that if the connections between the linkages are properly designed to preserve the integrity of the structure as well as to enable transmission of the signal through the different components, transition waves propagate through the entire structure and transform the initial architecture. Furthermore, the propagation of transition waves has also been demonstrated in 2D multistable elastic kirigami sheets[43] [Fig. 1(l)]. While homogeneous architectures result in constant-speed transition fronts, topological defects can be introduced to manipulate the pulses and redirect or pin transition waves, as well as to split, delay, or merge propagating wave fronts.

The results discussed in this section point to the rich dynamic responses of flexMMs. However, in order to enable such interesting behaviors the geometry of flexMM has to be carefully chosen. As such, it is crucial for the advancement of the field to develop models that can accurately predict these nonlinear behaviors and their dependency from geometric parameters and loading conditions.

III. MODELING THE NONLINEAR DYNAMIC RESPONSE OF FLEXMM

Discrete models have traditionally played an important role in unraveling the dynamic response of structures. Networks of point masses connected by linear springs have been routinely used to understand the propagation of linear waves in solid media.[77] Furthermore, by introducing nonlinear springs, these models have also enabled investigation of nonlinear waves in engineered media, including granular systems[27–32] and mass-spring lattices.[51] In recent years, discrete models have also proven useful to describe the nonlinear dynamic response of flexMM,[33–36,38,39,42,50,51,61,78–82] as they typically comprise stiffer elements connected by flexible hinges. The stiffer elements are modeled as rigid plates, whereas the response of the hinges is captured using a combination of rotational and longitudinal springs [see Fig. 2(a)]. Note that, since the rotation of the stiff elements plays a crucial role in flexMM, the rotational degrees of freedom of the rigid bodies play an important role in these models. For a typical 2D flexMM, three degrees of freedom (DOFs) are assigned to the ith rigid element: the displacement in x direction, u_i, the displacement in y direction, v_i, and rotation around the z axis, θ_i [see Fig. 2(b)]. Using these definitions, the equations of motion for the i-th rigid element are given by

$$m_i\ddot{u}_i = \sum_{p=1}^{N_{vi}} F^u_{i,p}, \quad m_i\ddot{v}_i = \sum_{p=1}^{N_{vi}} F^v_{i,p}, \quad \text{and} \quad J_i\ddot{\theta}_i = \sum_{p=1}^{N_{vi}} M^\theta_{i,p}, \tag{1}$$

where m_i and J_i are its mass and moment of inertia, respectively,

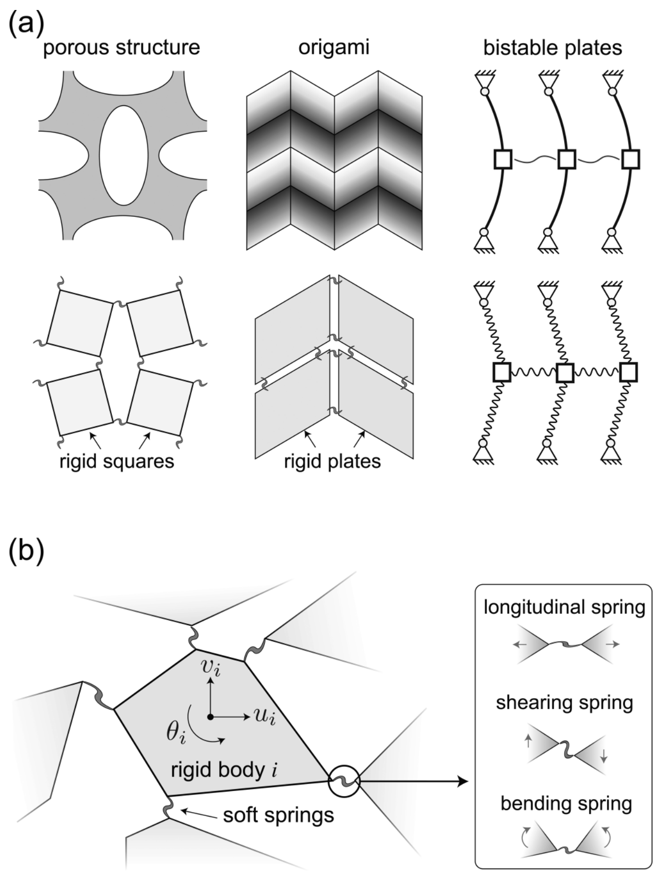

FIG. 2. Modeling the nonlinear dynamic response of FlexMM. (a) Schematics of three classes of flexMM and their corresponding discrete model. (b) The discrete models typically comprise networks of rigid bodies connected by a combination of rotational and longitudinal linear springs.

and N_{vi} denotes its number of vertices. Moreover, $F^u_{i,p}$ and $F^v_{i,p}$ are the forces along the x and y directions generated at the pth vertex of the ith units unit by the springs and $M^\theta_{i,p}$ represent the corresponding moment. Note that these forces can be expressed as a function of the DOFs of neighboring elements and are typically calculated assuming linear springs. Unlike typical mass-spring models previously used to investigate nonlinear waves, linear springs are sufficient to capture the dynamic response of flexMM, since the nonlinear behavior comes mainly from geometry. Equation (1) can subsequently be numerically integrated to obtain the dynamic response of the system. Importantly, these models provide a direct relation to the geometry of flexMMs, thus providing essential insights into their dynamic response.

For the special case of planar waves with characteristic wavelengths much larger than the unit cells, analytical solutions can also be obtained by taking the continuum limit of the discrete equations of motion.[33–37,51,61,78] There have been several examples of this: the response of an array of magnetically coupled bistable plates can be captured by a nonlinear Schrödinger equation;[51,78] the response of a flexMM based on the rotating-square mechanism can be captured by the nonlinear Klein–Gordon equation;[33–35,61] the response of a chain of buckled beams follows the Boussinesq equation;[36] and the dynamics of origami chains have been found to follow a Korteweg–de Vries equation.[37] Depending on the specific geometries of the flexMM and the driving input, these equations, when fully integrable, yield analytical solutions describing solitons,[33–35,78] rarefaction solitons,[36,37] and topological solitons.[61,78] Interestingly, these solutions provide a direct relation of these phenomena to the geometrical parameters of flexMMs. Therefore, they not only allow interpretation of the experimentally and numerically observed phenomena but also provide opportunities for the rational design of flexMM with targeted nonlinear dynamic responses.

For example, for a metamaterial based on the rotating-square mechanism by taking the continuum limit of Eq. (1), retaining nonlinear terms up to the third order and introducing the traveling wave coordinate $\zeta = x\cos\phi + y\sin\phi - ct$ (where x and y are the Cartesian coordinates, t indicates time, and ϕ and c represent direction and velocity of the propagating planar wave) in the governing equations of motion, it is found that the propagation of large amplitude planar waves is described by a nonlinear Klein–Gordon equation of the form[33–35,61]

$$\frac{\partial^2\theta}{\partial\zeta^2} = C_1\theta + C_2\theta^2 + C_3\theta^3 + O(\theta^4), \tag{2}$$

where C_1, C_2, and C_3 are parameters that depend on the geometry of the flexMM and the flexibility of its hinges and can, therefore, be tailored by tuning the metamaterial design. Equation (2) admits well-known solitary wave solutions of the form[34]

$$\theta(\zeta) = \frac{1}{D_1 \pm D_2\cosh(\zeta/W)}, \tag{3}$$

with

$$D_1 = -\frac{C_2}{3C_1}, \quad D_2 = \sqrt{\frac{C_2^2}{9C_1^2} - \frac{C_3}{2C_1}}, \quad \text{and} \quad W = \frac{1}{\sqrt{C_1}}. \tag{4}$$

Equation (3), depending on the sign of D_1 and D_2, captures different types of stable nonlinear pulses, including solitons, rarefaction solitons,[34] and topological solitons.[61] Moreover, it is important to note that under the assumption of traveling wave coordinates, other types of governing nonlinear equations, including the Boussinesq equation[36] and Korteweg–de Vries equation,[50] can be transformed into a nonlinear Klein–Gordon equations, making Eq. (2) quite general.

Finally, energy balance considerations have also proven useful to predict the characteristics of topological solitons propagating in dissipative media.[76,83] Specifically, it has been shown that the wave speed can be estimated by balancing the total transported kinetic energy, the difference between the higher and lower energy wells for the asymmetric elements, and the energy dissipated.

Furthermore, energy considerations can also provide insight into topological solitons-based energy harvesting.[44]

IV. OUTLOOK

In summary, this Perspective paper has attempted to demonstrate that flexible mechanical metamaterials provide a rich platform to manipulate the propagation of nonlinear waves. We close this paper by identifying several challenges for future work.

Toward nonlinear periodic waves. Beyond pulse-like, large-amplitude waves with finite spatial and temporal extent (e.g., solitons and transition waves discussed in this work), it has been shown that rotating-square systems with either quadratic[84] or cubic nonlinearity[85] also support the propagation of *cnoidal waves* (see Fig. 3). Cnoidal waves are described by the Jacobi elliptic functions $\mathrm{dn}(\cdot|k)$, $\mathrm{sn}(\cdot|k)$, and $\mathrm{cn}(\cdot|k)$, where k is the elliptic modulus controlling the shape of the elliptical functions. These cnoidal wave solutions extend from linear waves (for $k \to 0$) to solitons (for $k \to 1$), while covering also a wide family of nonlinear periodic waves.[85] Furthermore, as depicted in Fig. 3, flexMM could also provide a laboratory test bed for the observation of other types of nonlinear waves,[86,87] including bright/dark solitons,[78] breathers, and rogue waves[88] (large-amplitude waves that suddenly appear/disappear unpredictably, typically observed in surface water waves[89]). Harmonic generation, frequency conversion,[90,91] and even actuation (via effects such as frequency-down conversion) are other exciting possibilities to be explored by the rational design of the nonlinear properties of flexMM. Typically in experiments, the periodic and modulated waves depicted in Fig. 3 are generated by a low-frequency shaker driving a boundary of the FlexMM in the range of 10 Hz–10 kHz. Other types of transducers, drivers, or actuators are conceivable depending on the dimensions of the microstructure, the frequency range of interest, and the desired effects.

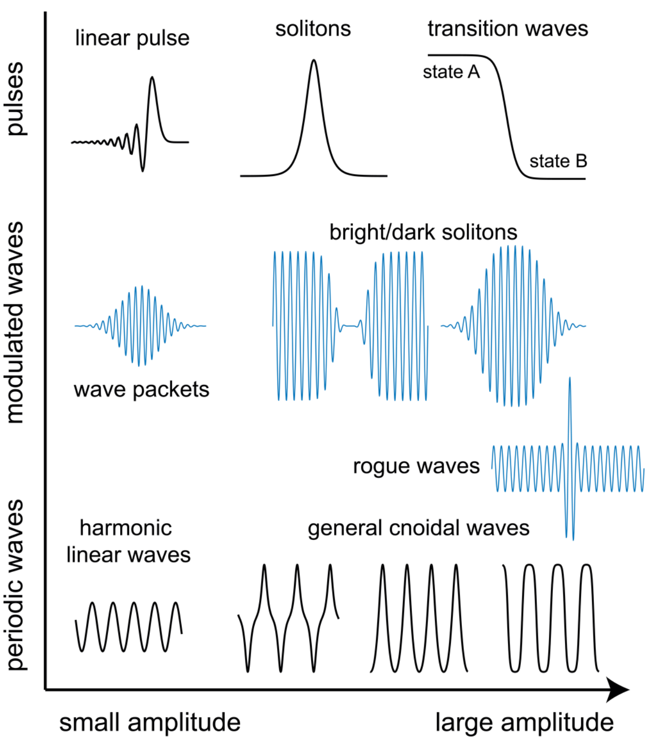

FIG. 3. A diagram showing different types of linear and nonlinear waves.

New flexMM designs. So far, the nonlinear dynamic response of a limited number of flexMM designs has been investigated. FlexMM based on origami, kirigami, tensegrity structures, and rotating-units other than squares (e.g., triangles or hexagons) may provide additional opportunities to manipulate the propagation of nonlinear waves. Also, the nonlinear dynamic responses of three-dimensional architectures remain largely unexplored and may open new avenues for wave management.

Going beyond periodic systems. While most previous studies have focused on the propagation of nonlinear pulses in periodic and homogeneous structures, new opportunities may arise when investigating the interactions of large-amplitude waves with free surfaces, inhomogeneous structures, and sharp interfaces. How do the nonlinear pulses propagate along free surfaces? What is the effect of internal interfaces on soliton propagation? How do other spatial variations such as gradients in initial angle, gradients in mass, or gradients in the stiffness of the hinges affect the propagation of waves through the material? Can these be used to steer a beam or otherwise affect an incident plane wave? All these questions remain unanswered.

Targeted nonlinear dynamical responses. While the focus so far has been on the development of tools to predict and characterize the propagating nonlinear waves in flexMM, an important question that is still unanswered is: How should one design the structure, including unit cell geometry, inhomogeneities such as gradients and interfaces, etc., to enable a target dynamic response? Target dynamic responses may include highly efficient damping for impact mitigation; optimal wave guiding (i.e., optimal energy confinement and propagation along a determined path); and lensing of solitons for optimal energy concentration. To allow the automated design of flexMM architectures that are optimal for achieving a specified set of target dynamic properties, one could couple discrete models with machine learning algorithms, such as neural networks and deep learning.

Control of nonlinear waves on the fly. Since the characteristics (i.e., shape, velocity, and amplitude) of nonlinear waves propagating through FlexMM can be tuned by varying the nonlinear response of the underlying medium, which can be effectively altered by (locally) deforming the metamaterial, we envision that the applied deformation could be a powerful tool to manipulate the pulses. Local deformations applied to the FlexMM could provide a mechanism to change the characteristics as well as the path of the propagating pulses on the fly. This could provide opportunities for time-space modulation of the propagating pulses,[82] real-time control of waves, and tunable non-reciprocal transmission.[92] Furthermore, since collisions of solitons in flexMM may result in anomalous interactions that provide opportunities to remotely detect, change, or eliminate high-amplitude signals and impacts,[50] we envision the use of collisions to pave new ways toward the advanced control of large amplitude mechanical pulses.

Reconfigurability via transition waves. As described in Sec. II C, the propagation of topological solitons (transition waves) in multi-stable flexMM can reconfigure all or part of the sample. Since this reconfiguration can be initiated even by a localized, weak impulse, a number of practical applications become possible. These include locomotion or propulsion in soft robotics,[39] precise and repeatable actuation,[24] and the reconfigurable devices mentioned earlier.[42,43,45] Next steps could include the use of inverse design tools and the controlled use of localized defects to achieve control of the propagation path or velocity of transition waves, enabling more complex actuation and targeted shape changes.

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DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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